

Anomalous Condensates and the Equivalence Theorem^{*}

William B. Kilgore

*Theoretical Physics Group
Lawrence Berkeley Laboratory
and
Department of Physics
University of California
Berkeley, California 94720*

A recently published report has called into question the validity of the equivalence theorem in dynamically broken gauge theories in which the fermions making up the symmetry breaking condensate lie in an anomalous representation of the broken gauge group. Such a situation can occur if the gauge anomaly is cancelled by another sector of the theory. Using the example of the one family Standard Model without scalar Higgs structure, we analyze a low energy effective theory which preserves the symmetries of the fundamental theory and demonstrate the validity of the equivalence theorem in this class of models.

^{*} This work was supported by the Director, Office of Energy research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Disclaimer

This document was prepared as an account of work sponsored by the United States Government. Neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof of The Regents of the University of California and shall not be used for advertising or product endorsement purposes.

Lawrence Berkeley Laboratory is an equal opportunity employer.

1. Introduction

Recently, Donoghue and Tandean[1] have discussed models of dynamical gauge symmetry breaking in which the fermions that participate in the symmetry breaking condensate lie in an anomalous representation of the broken symmetry. To preserve the gauge symmetry in the quantized theory there must be no net gauge anomaly.[2] Therefore there must also be additional fermions which cancel the gauge anomaly but do not take part in the condensate. They conclude that the equivalence theorem,[3] [4] which relates the scattering amplitudes of high-energy ($E \gg M$) longitudinal gauge bosons to those of the unphysical would-be Goldstone bosons, is invalid in such a theory. Specifically, they find that there is a non-zero scattering amplitude for the production of would-be Goldstone bosons via the Wess-Zumino anomaly interaction, while the production of gauge bosons, longitudinal or otherwise, through fermion triangle diagrams is forbidden by the anomaly cancellation condition.

This result would be quite disturbing, since the equivalence theorem is proven[3] to follow from the BRS[5] identities of the theory.* In this paper, we resolve the apparent paradox by showing that the anomaly cancellation condition does not imply that the production of gauge bosons via fermion triangle diagrams must vanish. Anomaly cancellation guarantees gauge invariance and causes the mass-independent pieces of the triangle diagrams to cancel one another. The mass dependent pieces are not constrained to cancel and in general do not cancel except when the fermion masses are equal. When the interactions of the would-be Goldstone bosons are properly formulated (*i.e.* such that the symmetries of the full theory, such as gauge invariance, are conserved), would-be Goldstone boson production is consistent with gauge boson production so that the equivalence theorem is indeed valid in anomalous condensate models.

2. A Toy Model

Let us carefully construct a simple theory that accomplishes spontaneous gauge symmetry breaking via an anomalous condensate. We consider a theory with the gauge structure of the Standard Model, a single family of fermions, and nothing else. In particular, there is no scalar Higgs sector to break the electroweak symmetry and give masses to the

* The existing proofs of the equivalence theorem are all in the context of a Higgs theory, but the proof can be carried over to a low energy effective field theory.[6]

gauge bosons or the fermions. Thus, the electroweak symmetry is broken dynamically, as in technicolor models, by the spontaneous chiral symmetry breakdown of QCD.[7]

The Lagrangian for our theory is:

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}G^{a\mu\nu}G_{\mu\nu}^a - \frac{1}{4}W^{i\mu\nu}W_{\mu\nu}^i - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ & + \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R + \bar{l}_L i \not{D} l_L + \bar{e}_R i \not{D} e_R,\end{aligned}\tag{2.1}$$

where $G_{\mu\nu}^a$, $W_{\mu\nu}^i$ and $B_{\mu\nu}$ are the QCD, $SU(2)_L$, and hypercharge field strength tensors, respectively and

$$\begin{aligned}q &= \begin{pmatrix} u \\ d \end{pmatrix}, & l &= \begin{pmatrix} \nu \\ e \end{pmatrix}, \\ D_\mu &= \partial_\mu - ig_3 G_\mu^a \frac{\lambda^a}{2} - ig_2 W_\mu^i \frac{\sigma^i L}{2} - ig_1 B_\mu Y\end{aligned}\tag{2.2}$$

where λ^a acts only on the color triplet quarks, L is the left chirality projection operator, $L = \frac{1-\gamma_5}{2}$, and Y represents the usual Standard Model hypercharge quantum number for each fermion species.

Being massless, the quarks exhibit an exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry. At scale Λ_χ , a fermion condensate forms with a non-zero vacuum expectation value,

$$\langle \bar{u}u + \bar{d}d \rangle \neq 0,\tag{2.3}$$

which breaks the $SU(2)_L \otimes SU(2)_R$ symmetry down to $SU(2)_V$. Associated with this symmetry breakdown are three Goldstone bosons, the pions, one for each broken symmetry generator.

The pions transform under a nonlinear representation of $SU(2)_L \otimes SU(2)_R$, forming a (linear) triplet representation of the unbroken $SU(2)_V$ symmetry, but transforming nonlinearly under the broken axial $SU(2)_A$. All such nonlinear realizations are physically equivalent[8] and their interactions are described by the so-called chiral Lagrangian, which may be written as

$$\mathcal{L}_{GB} = \frac{F_\pi^2}{4} \text{Tr} \left(D^\mu \Sigma^\dagger D_\mu \Sigma \right) + \dots\tag{2.4}$$

where

$$\begin{aligned}\Sigma &= \exp \left(i \frac{\boldsymbol{\sigma} \cdot \boldsymbol{\pi}}{F_\pi} \right), \\ D_\mu \Sigma &= \partial_\mu \Sigma - ig_2 W_\mu^i \frac{\sigma^i}{2} \Sigma + ig_1 B_\mu \Sigma \frac{\sigma^3}{2},\end{aligned}\tag{2.5}$$

the π^i 's are the pion fields, the σ^i 's are the Pauli matrices, and F_π , called the pion decay constant, is the strength of the coupling of the pions to the axial $SU(2)_A$ currents,

$$\langle 0 | J_\mu^i | \pi^j(q) \rangle = i F_\pi q_\mu \delta^{ij}. \quad (2.6)$$

Under $SU(2)_L \otimes SU(2)_R$ transformations, Σ transforms as

$$\Sigma \longmapsto L \Sigma R^\dagger. \quad (2.7)$$

The ellipses in equation (2.4) indicate terms that involve higher covariant derivatives and are assumed to be small at low energy.

In addition to pion kinetic terms and multi-pion interaction terms, we find that \mathcal{L}_{GB} includes vector boson mass terms and gauge boson – pion mixing terms which are exactly those found in the Standard Model with F_π taking the place of the Higgs vacuum expectation value v and the pions serving as the would-be Goldstone bosons. Thus the pions are not physical degrees of freedom, but are absorbed into the gauge bosons via the Higgs mechanism. Since the vector mass terms have changed in scale only ($v \rightarrow F_\pi$), the Weinberg angle, parameterizing the mixing of W^3 and B into γ and Z as well as the W to Z mass ratio, is the same as in the Standard Model.

3. Chiral Quarks

The quarks participate in the condensate and acquire mass through their coupling to it. Below the chiral symmetry breaking scale, Λ_χ , the quark Lagrangian may be written[9] [10] as

$$\mathcal{L}_q = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - m_Q (\bar{q}_L \Sigma q_R + \bar{q}_R \Sigma^\dagger q_L) + \dots \quad (3.1)$$

Upon expanding Σ in powers of π , one sees that the zeroth order term is the mass term for the quarks. This mass, m_Q , is called the constituent mass of the quarks and is expected to be of the same order of magnitude as the chiral symmetry breaking scale Λ_χ .

The Lagrangian in equation (3.1) gives us a perfectly good description of the low energy quark interactions, but does not explicitly exhibit decoupling of the massive quarks from very low energy ($s \ll m_Q^2$) processes. For instance, the coupling of two vector currents to the pseudoscalar pions via quark triangle diagrams (VVP triangles), does not vanish for $s \ll m_Q^2$, even in the limit that m_Q approaches infinity.[11] Gauge invariance forbids quark decoupling; the leptons carry an anomalous gauge dependence which is exactly cancelled

by the anomalous gauge dependence of the quarks. If the quarks decoupled, there would be nothing to cancel the gauge dependence of the leptons. It is possible to write the Lagrangian in a form such that the quarks can be decoupled,[12] but there are subtleties involved and we will examine the procedure carefully. (We follow here the work of Manohar and Moore.[10])

The anomalous gauge dependence of the quark Lagrangian in equation (3.1) is tied to the fact that the left- and right-handed quarks transform independently under $SU(2)_L \otimes SU(2)_R$ rotations:

$$q_L \longmapsto Lq_L, \quad q_R \longmapsto Rq_R. \quad (3.2)$$

However, it is possible to define a new basis for the quarks so that both left- and right-handed fields transform in the same way. To that end, we define the matrix ξ as

$$\xi^2 \equiv \Sigma. \quad (3.3)$$

Under chiral rotations, ξ transforms as

$$\xi \longmapsto L\xi U^\dagger(x) = U(x)\xi R^\dagger, \quad (3.4)$$

where $U(x)$ is a unitary matrix implicitly defined by the above equations. Using ξ one may rotate the quarks to a new basis Q , defined by

$$Q_L = \xi^\dagger q_L, \quad Q_R = \xi q_R \quad (3.5)$$

which transform as

$$Q_{L,R} \longmapsto U(x)Q_{L,R}. \quad (3.6)$$

We define vector and axial vector fields as

$$\mathcal{V}_\mu = \frac{1}{2} (\xi(D_\mu \xi^\dagger) + \xi^\dagger(D_\mu \xi)), \quad \mathcal{A}_\mu = \frac{i}{2} (\xi(D_\mu \xi^\dagger) - \xi^\dagger(D_\mu \xi)), \quad (3.7)$$

where

$$D_\mu \xi \equiv \partial_\mu \xi - ig_2 W_\mu^i \frac{\sigma^i}{2} \xi - ig_1 B_\mu \frac{1}{6} \xi, \quad D_\mu \xi^\dagger \equiv \partial_\mu \xi^\dagger - ig_1 B_\mu \left(\frac{1}{6} + \frac{\sigma^3}{2} \right) \xi^\dagger. \quad (3.8)$$

which transform as

$$\mathcal{V}_\mu \longmapsto U\mathcal{V}_\mu U^\dagger + U\partial_\mu U^\dagger, \quad \mathcal{A}_\mu \longmapsto U\mathcal{A}_\mu U^\dagger. \quad (3.9)$$

In this notation, the Goldstone boson Lagrangian is written as

$$\mathcal{L}_{GB} = \frac{F_\pi^2}{4} \text{Tr}(\mathcal{A}^\mu \mathcal{A}_\mu) + \dots \quad (3.10)$$

and the quark Lagrangian as

$$\mathcal{L}_Q = \bar{Q} (i \not{D}^Q - m_Q) Q + \bar{Q} \mathcal{A} \gamma_5 Q + \dots,$$

where

$$\mathcal{D}_\mu^Q Q = \left(\partial_\mu - i g_3 G_\mu^a \frac{\lambda^a}{2} + \mathcal{V}_\mu \right) Q. \quad (3.11)$$

The Lagrangian in equation (3.1) and the above expression for \mathcal{L}_Q are not equivalent. Since Q_L and Q_R transform in the same way, \mathcal{L}_Q is not anomalous under local chiral $SU(2)_L \otimes SU(2)_R$ rotations and since electroweak gauge rotations are simply a subset of these local chiral rotations, \mathcal{L}_Q has no anomalous gauge dependence. Somehow, in changing quark bases, we seem to have lost the anomalous gauge dependence that we need so that the complete theory is gauge invariant. This apparent paradox arises because the change of variables from q to Q changes the measure of the path integral, requiring a non-trivial Jacobian factor. This Jacobian factor carries the anomalous gauge dependence.

A change in the measure of the path integral is difficult to incorporate in a perturbative expansion. So, just as we move the gauge fixing condition and the Fadeev-Popov determinant out of the measure of the path integral and into the effective Lagrangian, here too we add a new term to the Lagrangian which exactly compensates for the change in measure induced by the quark rotation. This compensating term is the Wess-Zumino anomaly term,[13] which for the case at hand is

$$\begin{aligned} \mathcal{L}_{WZ} = \frac{1}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \left\{ \mathbf{B}_\mu \partial_\nu \mathbf{B}_\rho \text{Tr} \left[\frac{\sigma^3}{2} (\Sigma^\dagger (\mathbf{W}_\sigma + \mathbf{L}_\sigma) \Sigma - \mathbf{W}_\sigma) \right] \right. \\ \left. + \partial_\mu \mathbf{B}_\nu \text{Tr} (\mathbf{W}_\rho \mathbf{L}_\sigma) - \frac{1}{3} \mathbf{B}_\mu \text{Tr} (\mathbf{L}_\nu \mathbf{L}_\rho \mathbf{L}_\sigma) \right\}, \end{aligned} \quad (3.12)$$

with $\mathbf{L}_\mu = (\partial_\mu \Sigma) \Sigma^\dagger$, $\mathbf{W}_\mu = -i g_2 W_\mu^i \frac{\sigma^i}{2}$ and $\mathbf{B}_\mu = -i g_1 B_\mu$.

Thus, our complete low-energy Lagrangian is:

$$\begin{aligned} \mathcal{L} = -\frac{1}{4} W^{i\mu\nu} W_{\mu\nu}^i - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{F_\pi^2}{4} \text{Tr}(\mathcal{A}^\mu \mathcal{A}_\mu) + \mathcal{L}_{gf} + \bar{l}_L i \not{D} l_L + \bar{e}_R i \not{D} e_R \\ + \bar{Q} (i \not{D}^Q - m_Q) Q + \bar{Q} \mathcal{A} \gamma_5 Q + \mathcal{L}_{WZ} + \dots, \end{aligned} \quad (3.13)$$

where \mathcal{L}_{gf} contains the gauge fixing and Fadeev-Popov ghost terms necessary for gauge field quantization. (We assume, of course, that electroweak gauge fixing is accomplished through a “ ξ -gauge” condition,[14] $\mathcal{L}_{gf} = \frac{1}{2\xi} (\partial_\mu A^\mu - \xi M\pi)^2$, in which we know how to prove the equivalence theorem.) The quarks now have derivative couplings to the pions and thus the VVP triangles connecting pions to vector currents in the q basis are replaced by Vector – Vector – Axial vector (VVA) triangles and gauge – Goldstone contact interactions in the Wess-Zumino term. The VVA triangles, and all other terms explicitly involving quarks, decouple as the quark mass is taken to infinity. The Wess-Zumino term does not explicitly involve the quarks and does not decouple. It cannot decouple because it is not gauge invariant. In the q basis, the quarks cannot decouple because they must cancel the anomalous gauge dependence of the leptons. In the Q basis, the Wess-Zumino term balances the gauge dependence of the leptons; the Q quarks are gauge invariant and there is no obstacle to their decoupling. We can directly integrate out the quarks in the Q basis but not in the q basis.

4. The Equivalence Theorem

The simplest process involving the anomaly which one might investigate is the well known decay $\pi^0 \longrightarrow \gamma\gamma$, and its analog through the equivalence theorem $Z_L \longrightarrow \gamma\gamma$. This process, however, is not an appropriate test of the equivalence theorem which is explicitly formulated to be valid for high energy processes ($E \gg M_{W,Z}$), and relies upon the fact that the longitudinal polarization vector for fast moving gauge bosons is approximately proportional to the momentum of the particle. This approximation is clearly invalid in the rest frame.

Let us therefore examine a process where we can produce energetic gauge bosons such as $\bar{\nu} + \nu \longrightarrow Z_L + \gamma$. This process provides a particularly clean example to study as the more realistic process $e^+ + e^- \longrightarrow Z_L + \gamma$ is also more complicated because of $Z - \gamma$ interference.

The relevant diagrams for the q and Q bases are shown in figures 1 and 2 respectively. (Implicitly included are the triangle diagrams with the fermions propagating in the opposite direction around the loop.) In the q basis, there is no Wess-Zumino term, so the VVP triangle of figure 1c completely describes $\pi^0\gamma$ production, while in the Q basis $\pi^0\gamma$

production results from the sum of the VVA triangle of Figure 2c and the Wess-Zumino interaction of Figure 2d. The amplitude for $\pi^0\gamma$ production via the Wess-Zumino interaction is

$$\mathcal{M}_{\pi\gamma}^{WZ} = \frac{ieg_2^2}{64\pi^2 F_\pi} \frac{(1 - 4\sin^2\theta_W)}{\cos^2\theta_W} \frac{\bar{\nu}\gamma_\lambda(1 - \gamma_5)\nu}{s - M_Z^2} \varepsilon^{\alpha\beta\nu\lambda} p_{1\alpha} p_{2\beta} \epsilon_\nu^\gamma(p_2), \quad (4.1)$$

where ϵ^γ is the photon polarization vector. The other amplitudes, computed in the limit that $\epsilon_\mu^{Z_L}(p_1) \approx \frac{p_{1\mu}}{M_Z} + \mathcal{O}\left(\frac{M_Z}{E}\right)$, are:

$$\begin{aligned} \mathcal{M}_{\pi\gamma}^Q &= -\mathcal{M}_{\pi\gamma}^{WZ} \left[1 - 2 \int_0^1 dx \int_0^{1-x} dy \frac{m_Q^2}{(y^2 - y) M_Z^2 - xy(s - M_Z^2) + m_Q^2} \right] \\ \mathcal{M}_{\pi\gamma}^q &= \mathcal{M}_{\pi\gamma}^{WZ} + \mathcal{M}_{\pi\gamma}^Q \\ \mathcal{M}_{Z\gamma}^l &= -i\mathcal{M}_{\pi\gamma}^{WZ} + \mathcal{O}\left(\frac{M_Z}{E}\right) \\ \mathcal{M}_{Z\gamma}^Q &= -i\mathcal{M}_{\pi\gamma}^Q + \mathcal{O}\left(\frac{M_Z}{E}\right) = \mathcal{M}_{Z\gamma}^q \end{aligned} \quad (4.2)$$

where x and y are Feynman parameters. Up to a phase,

$$\mathcal{M}_{\pi\gamma}^Q + \mathcal{M}_{\pi\gamma}^{WZ} = \mathcal{M}_{\pi\gamma}^q = \mathcal{M}_{Z\gamma}^l + \mathcal{M}_{Z\gamma}^Q + \mathcal{O}\left(\frac{M_Z}{E}\right), \quad (4.3)$$

exactly as the equivalence theorem asserts.

There are several features worth noticing. First, by explicit calculation, $\mathcal{M}_{\pi\gamma}^Q + \mathcal{M}_{\pi\gamma}^{WZ}$ is exactly equal to $\mathcal{M}_{\pi\gamma}^q$. This is as it must be since the q and Q formulations are equivalent. Second, the validity of the equivalence theorem is independent of the constituent quark mass m_Q . We see this most clearly in the Q basis where we find that gauge boson production via quark triangles is equivalent to would-be Goldstone production via quark triangles, and gauge boson production via lepton triangles is equivalent to would-be Goldstone production via the Wess-Zumino interaction. The quark triangles for gauge and would-be Goldstone boson production have identical mass dependence while the lepton triangles and the Wess-Zumino interaction are independent of the quark mass.

Finally, for $M_Z^2 \ll s \ll m_Q^2$, the quark triangles, $\mathcal{M}_{Z\gamma}^Q$ and $\mathcal{M}_{\pi\gamma}^Q$ are suppressed relative to the lepton triangles $\mathcal{M}_{Z\gamma}^l$ and the Wess-Zumino term $\mathcal{M}_{\pi\gamma}^{WZ}$ respectively, providing nonvanishing production amplitudes for $Z_L\gamma$ and $\pi^0\gamma$. For $s \gg m_Q^2$, however, the quark triangles are not suppressed relative to the lepton triangles and the Wess-Zumino term and tend to cancel them so that the production amplitudes are quite small. For example, at an

intermediate energy $M_Z^2 \ll s \ll m_Q^2$, the cross section for $Z_L \gamma$ production is approximately s independent,

$$\sigma_{Z_L \gamma} (M_Z^2 \ll s \ll m_Q^2) = \frac{\alpha \alpha_W^2 (1 - 4 \sin^2 \theta_W)}{3072 \pi^2 F_\pi^2 \cos^4 \theta_W} \left(1 - \frac{s}{6m_Q^2}\right), \quad (4.4)$$

while at high energy, $s \gg m_Q^2$, the cross section falls rapidly with s ,

$$\sigma_{Z_L \gamma} (s \gg m_Q^2) = \frac{\alpha \alpha_W^2 (1 - 4 \sin^2 \theta_W)}{3072 \pi^2 F_\pi^2 \cos^4 \theta_W} \frac{\pi^4 m_Q^4}{s^2} \left(1 + \frac{1}{\pi^2} \left(\ln \frac{s}{m_Q^2}\right)^2\right)^2. \quad (4.5)$$

Let us now return to reference [1]. Donoghue and Tandean compute $\pi^0 \gamma$ production exclusively through the Wess-Zumino interaction, while finding that $Z_L \gamma$ production vanishes because the quark triangles exactly cancel the lepton triangles. From equation (4.2), we see that their computation of $\pi^0 \gamma$ production is a valid low energy (and/or heavy quark) approximation, ($s \ll m_Q^2$ so that $|\mathcal{M}_{\pi\gamma}^Q| \ll |\mathcal{M}_{\pi\gamma}^{WZ}|$) while their result for $Z_L \gamma$ production is a valid high energy (and/or light quark) approximation. ($s \gg m_Q^2$ so that $\mathcal{M}_{Z\gamma}^Q \approx -\mathcal{M}_{Z\gamma}^l$) Each is valid within a particular energy regime, but they are never simultaneously valid.

5. Comments

One complication that we did not include, though we know it to be present in the real world, is that the strong QCD forces renormalize the hadronic part of the axial vector current so that the $\bar{Q} \mathcal{A}_{\gamma 5} Q$ term in equation (3.13) should enter with a coefficient g_A which could be determined experimentally. The introduction of g_A is the only change that needs to be made to the Lagrangian in the Q basis. Both $\mathcal{M}_{Z\gamma}^Q$ and $\mathcal{M}_{\pi\gamma}^Q$ are multiplied by g_A while the lepton couplings and the Wess-Zumino interaction are unchanged so that the equivalence theorem remains intact. More complicated modifications must be made in the q basis,[10] but the conclusions are of course the same.

Another complication that we have not addressed involves enlarging the scalar sector. This would happen if more than one electroweak doublet were to participate in the condensate. In this case, we would again find the equivalence theorem to be valid. As discussed above, in the Q basis quark triangle production of high energy gauge bosons is manifestly equivalent to quark triangle production of would-be Goldstone bosons. Since the strength of the Wess-Zumino term is determined by the strength of the quark anomaly[13] which, because of anomaly cancellation is equal in magnitude to the lepton anomaly, there is again equivalence between gauge boson production via lepton triangles and would-be Goldstone production via the Wess-Zumino interaction. Thus, the overall equivalence is maintained.

6. Conclusions

The equivalence theorem is a direct consequence of the BRS symmetry of a spontaneously broken gauge theory. Since BRS symmetry is the special form of the classical gauge symmetry which survives the quantization procedure, the equivalence theorem is a necessary consequence of gauge invariance. In this letter, we have constructed an explicit example of how the equivalence theorem is satisfied even in the presence of an anomalous symmetry breaking condensate. Gauge invariance demands that the anomaly associated with the condensate be cancelled by some other part of the theory. Together, the two parts combine to preserve gauge invariance and in so doing guarantee the equivalence theorem.

In demonstrating the validity of the equivalence theorem, we have seen that anomalous condensate models permit gauge boson production through fermion triangle diagrams. It may be that there are ‘anomalous technicolor’ models for which this production mechanism has observable consequences.[15]

Acknowledgements

I would like to thank Marcus Luty, Greg Keaton and Michael Chanowitz for many helpful discussions.

This work was supported by the Director, Office of Energy research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

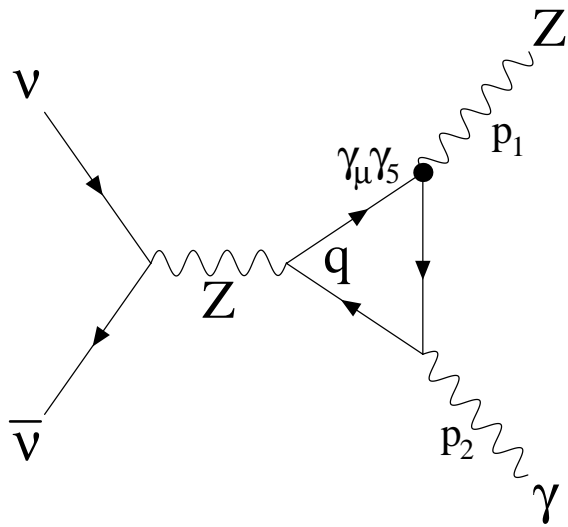
References

- [1] J.F. Donoghue and J. Tandean, Phys. Lett. B 301 (1993) 372.
- [2] D.J. Gross and R. Jackiw, Phys. Rev. D 6 (1972) 477.
C. Bouchiat, J. Illiopoulos and Ph. Meyer, Phys. Lett. B 38 (1972) 519.
- [3] M.S. Chanowitz and M.K. Gaillard, Nucl. Phys. B 261 (1985) 379.
G.J. Gounaris, R. Kögerler and H. Neufeld, Phys. Rev. D 34 (1986) 3257.
J. Bagger and C. Schmidt, Phys. Rev. D 41 (1990) 264.
H. Veltman, Phys. Rev. D 41 (1990) 2294.
W. Kilgore, Phys. Lett. B 294 (1992) 257, 1992.
H.J. He, Y.P. Kuang and X. Yi, Phys. Rev. Lett. 69 (1992) 2619.
H.J. He, Y.P. Kuang and X. Yi, TUIMP preprint TUIMP-TH-92/51
- [4] J.M. Cornwall, D.N. Levin, and G. Tiktopoulos, Phys. Rev. D 10 (1974) 1145.
C.E. Vayonakis, Lett. Nuovo Cim. 17 (1976) 383.
B.W. Lee, C. Quigg and H. Thacker, Phys. Rev. D 16 (1977) 1519.
D. Soper and Z. Kunst, Nucl. Phys. B 296 (1988) 253.
- [5] C. Becchi, A. Rouet and R. Stora, Comm. Math. Phys. B 261 (1975) 379.
- [6] W. Kilgore, in preparation.
- [7] M. Weinstein, Phys. Rev. D 7 (1973) 1854; Phys. Rev. D 8 (1973) 2511.
S. Weinberg, Phys. Rev. D 19 (1979) 1277.
L. Susskind, Phys. Rev. D 20 (1979) 2619.
- [8] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239.
C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.
- [9] A. Manohar and H. Georgi, Nucl. Phys. B 234 (1984) 189.
- [10] A. Manohar and G. Moore, Nucl. Phys. B 243 (1984) 55.
- [11] J. Steinberger, Phys. Rev. 76 (1949) 1180.
- [12] E. D'Hoker and E. Farhi, Nucl. Phys. B 248 (1984) 59,77.
- [13] J. Wess and B. Zumino, Phys. Lett. B 37 (1971) 95.
E. Witten, Nucl. Phys. B 223 (1983) 422.
W.A. Bardeen, Phys. Rev. 184 (1969) 1848.
K. Fujikawa, Phys. Rev. Lett. 42 (1979) 1195; Phys. Rev. D 21 (1980) 2848;
Phys. Rev. D 22 (1980) 1499; Phys. Rev. D 23 (1981) 2262.
B. Zumino, in: Relativity, Groups, and Topology II, eds. B.S. DeWitt and R. Stora
(North-Holland, Amsterdam, 1984) p.1291.
B. Zumino, Y.-S. Wu and A. Zee, Nucl. Phys. B 239 (1984) 477.
W.A. Bardeen and B. Zumino, Nucl. Phys. B 244 (1984) 421.
L. Alvarez-Gaumé and P. Ginsparg, Ann. Phys. 161 (1985) 423.
N.K. Pak and P. Rossi, Nucl. Phys. B 250 (1985) 279.

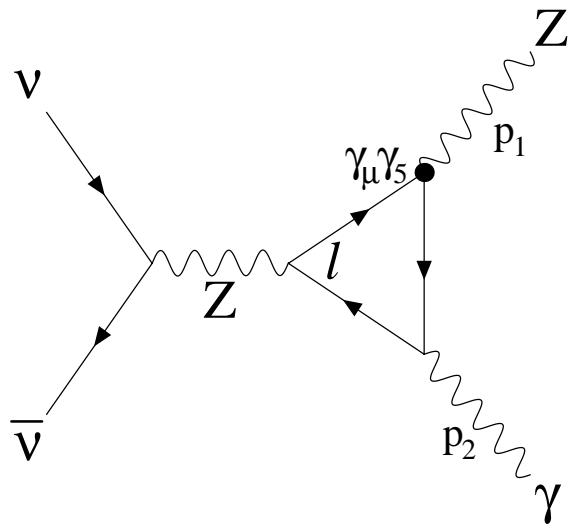
- [14] K. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6 (1972) 2923.
- [15] W. Kilgore, in preparation.

Figure Captions

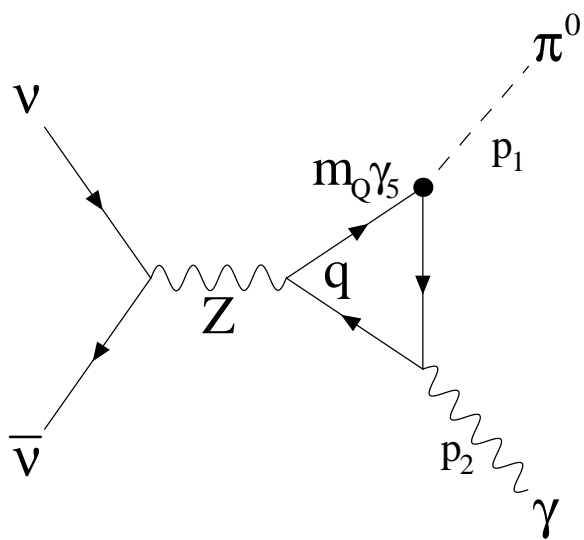
- Figure 1. Feynman diagrams for $Z_L\gamma$ and $\pi\gamma$ production with quarks in the q basis. The Dirac structure of the external Z_L / π couplings to the fermion triangles is indicated. The full vertex factors are: 1a) $\mp \frac{ig_2}{2\cos\theta_W}\not{p}\gamma_5 \approx \mp \frac{i}{F_\pi}\not{p}\gamma_5$ for $\begin{pmatrix} u \\ d \end{pmatrix}$ quarks; 1b) $\approx \frac{i}{F_\pi}\not{p}\gamma_5$; and 1c) $\pm \frac{m_Q}{F_\pi}\gamma_5$.
- Figure 2. Feynman diagrams for $Z_L\gamma$ and $\pi\gamma$ production with quarks in the Q basis. The Dirac structure of the external Z_L / π couplings to the fermion triangles is indicated. The full vertex factors are: 2a) $\mp \frac{ig_2}{2\cos\theta_W}\not{p}\gamma_5 \approx \pm \frac{i}{F_\pi}\not{p}\gamma_5$ for $\begin{pmatrix} u \\ d \end{pmatrix}$ quarks; 2b) $\approx \frac{i}{F_\pi}\not{p}\gamma_5$; and 2c) $\pm \frac{1}{F_\pi}\not{p}\gamma_5$. Figure 2d) represents $\pi\gamma$ production via the Wess-Zumino term.



1a



1b



1c

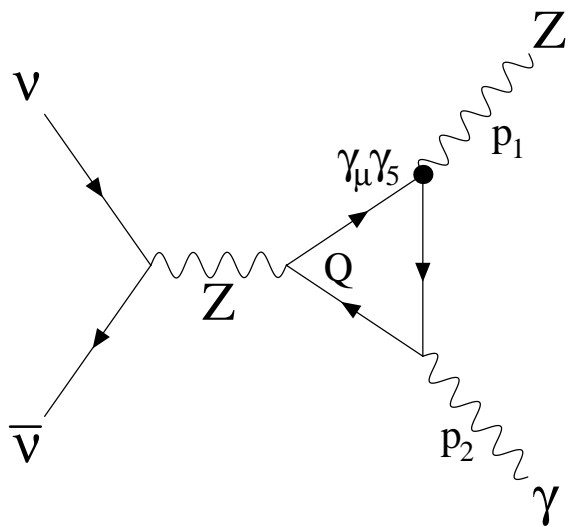
Figure 1

This figure "fig1-1.png" is available in "png" format from:

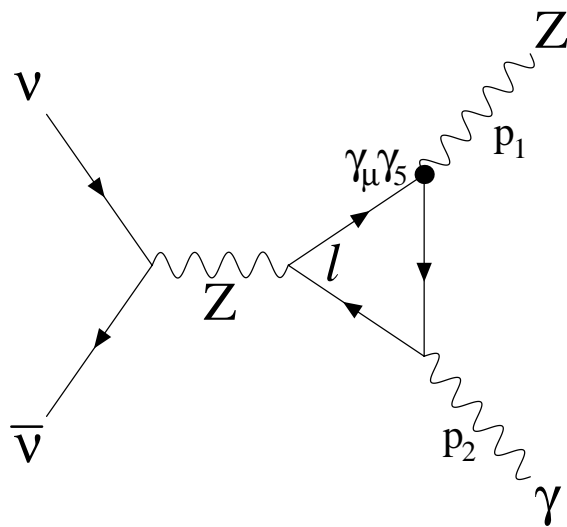
<http://arXiv.org/ps/hep-ph/9311379v1>

This figure "fig2-1.png" is available in "png" format from:

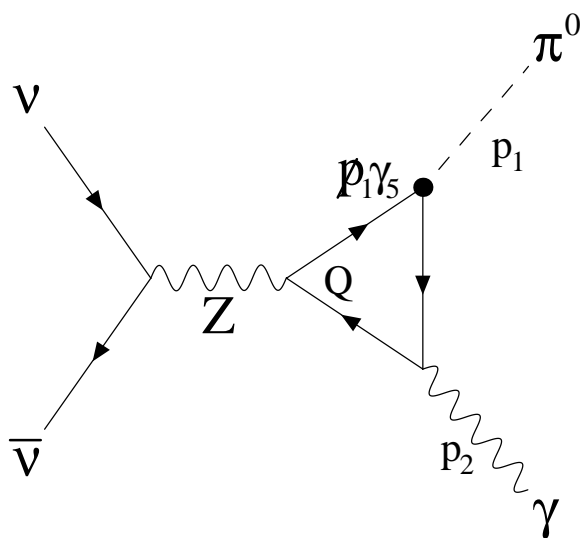
<http://arXiv.org/ps/hep-ph/9311379v1>



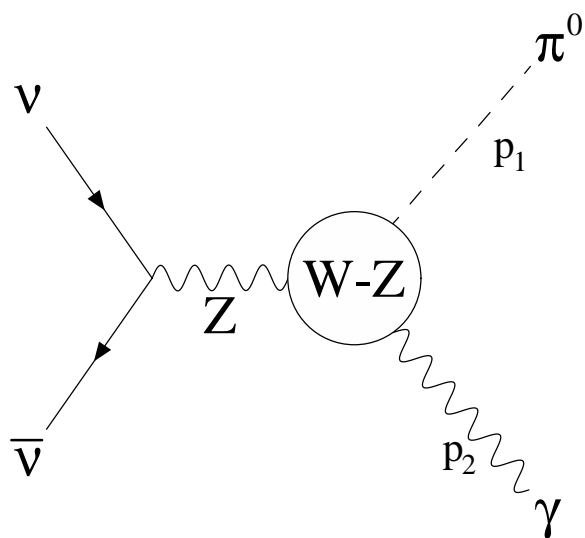
2a



2b



2c



2d

Figure 2